Welcome!

CS 103ACE Day 2 – 4/8/24

Pull up Slack and fill out the attendance question!

Agenda:

- Announcements & intros
- Direct proof practice
- Proof by contradiction
- Proof by contrapositive

Announcements

- Really important things from section 1:
  - Please come to section having watched the previous lecture
  - We won’t be working on the problem sets during section
  - My entire job is to help you do your best in this class!
    I believe in you :)
- New faces!

Proof Strategies

Today's learning goals:

- Build your toolkit for writing proofs
- Take the negation of a proposition
- Take the contrapositive of a proposition
- Practice writing proofs!

Problem 3a. Three Step Plan for Proofs

1. **What is this asking me to do?**
   - Write down all relevant definitions
   - (Optional: Come up with examples)
2. **What is the argument I'm going to make?**
   - The structure of the theorem → the structure of your proof
   - Write down your assume & want-to-show
     (Tip: use numbered lists)
3. **How do I explain my argument in a logical order?**
   - Apply a format from lecture
   - You may have to write "out of order": the logical order of facts might differ from how you came up with the proof

Proof Strategy: Proof by Contradiction

Prove X by showing that if X wasn't true, we get an impossible and silly outcome. The whole time, **we are still trying to prove X**, even though we start from assuming "not X".

- Step 1: Figure out what "not X" is.
  - Key tip: When X is an implication "if P, then Q", the negation of X is "P and not Q".
- Step 2: We don't have an explicit "want to show". We just assume not X and want to eventually find something fishy.
- Step 3: "Assume for the sake of contradiction that..." then state your assumption

Three ways to introduce variables

See the proofwriting checklist

- Reader picked: pick any value (possibly under certain constraints)
  - “Pick an **arbitrary** odd integer x”
  - “Let z be **any** natural number”
- Writer picked: give a specific value
  - “Let x be **5**”
  - “Pick z to be **{137}**”
- “Existentially picked”: when you know something exists, but you don’t know what it is
  - “Since x is odd, we know **there is a** number k where $x = 2k + 1$”
  - “We know that [a fuzzy unicorn] **exists**. Let y be [a fuzzy unicorn].”

Proof Strategy: Proof by Contrapositive

We are going to prove P → Q by proving (not Q) → (not P)

- Step 1: Figure out what P, Q, not P, and not Q are
- Step 2: Set up the proof like with any implication:
  - Assume (not Q), want to show (not P)
- Step 3: “We will prove the contrapositive:” then state the contrapositive

Post-section recommendations

- Problem Set 1
  - Start the problem set if you haven't already!
- Proofs practice:
  - Do the odd and even number exercises from Lecture 1. Send your proofs to me or post them on Ed.
  - Do ACE extra problems 2.2-2.4.
- Reminder: fill out the intro form to express preferences on office hours times.