1. Propositional Logic Review

Fill in this table with the negations from class. There are two negations for A $\leftrightarrow$ B.

Expression	English Translation	Negation of Expression
A $\wedge$ B		
A $\vee$ B		
$\neg$A		
A $\rightarrow$ B		
A $\leftrightarrow$ B		
$\top$		
$\bot$		

2. Negating Propositional Statements

We'll demonstrate "starting from the outside" when taking negations in class. Take the negation of the following statements:

a. A $\rightarrow$ $\neg$B

b. (C $\rightarrow$ D) $\wedge$ (D $\rightarrow$ C)

c. $\neg$Q $\rightarrow$ $\neg$P

d. ($\neg$X $\wedge$ Y) $\vee$ Z

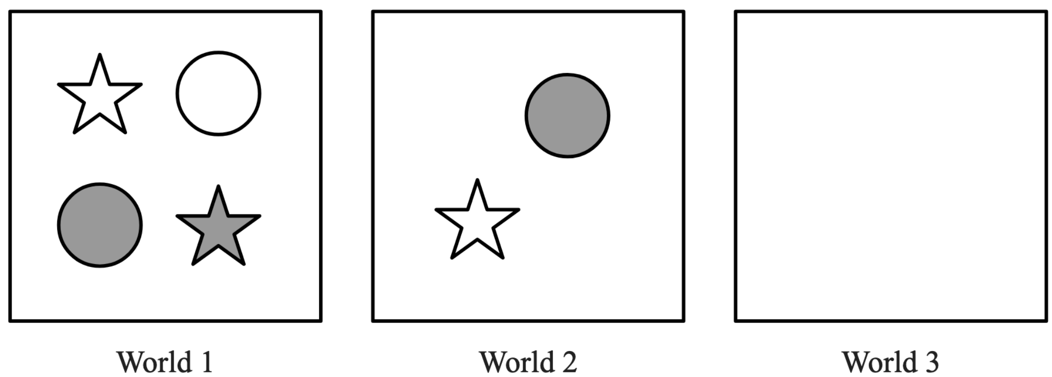
3. First-Order Logic I Review

Fill in this table:

Expression	English Translation
$\forall x.(P(x))$	
$\exists x.(P(x))$	
$\forall x.(A(x) \rightarrow B(x))$	
$\exists x.(A(x) \wedge B(x))$	

4. Evaluating First-Order Logic Statements

Let's say the predicate $Gray(x)$ is true if x is gray, $White(x)$ is true if x is white, $Star(x)$ is true if x is a star, and $Circle(x)$ is true if x is a circle. Consider the following three worlds:



For each of the following first-order logic statements, say whether it's true in each of the worlds.

Expression	World 1	World 2	World 3
$\exists x. (Circle(x))$	$\top$	$\top$	$\bot$
$\forall x. (Circle(x))$			
$\forall x. (White(x) \to Star(x))$			
$\forall x. (Star(x) \to White(x))$			
$\exists x. (Gray(x) \land Star(x))$			
$\exists x. (Gray(x) \lor Star(x))$			
$\forall x. (White(x) \leftrightarrow Circle(x))$			

5. Translating English to First-Order Logic

Translate each statement into first-order logic, given these predicates: $HasHat(x)$ says that x is wearing a hat and $Dog(x)$ says that x is a dog.

a. There is a dog.

b. All dogs wear hats.

c. There is a dog with a hat.

d. Some dogs don't wear hats.

e. Some dogs wear hats, but not all dogs wear hats.

6. First-Order Logic II Review

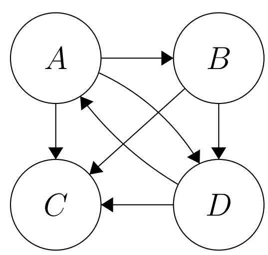
Fill in this table of the four basic forms:

Expression	English Translation	Negation of Expression
$\forall x.(A(x) \to B(x))$		
$\exists x.(A(x) \land B(x))$		
$\forall x.(A(x) \to \neg B(x))$		
$\exists x.(A(x) \land \neg B(x))$		

7. Evaluating Nested First-Order Logic Statements

For each statement, translate it into English, then decide whether it's true or false.

Interpersonal Dynamics: This diagram represents a set P of people named A, B, C and D. If there's an arrow from a person x to a person y, then person x loves person y. We'll denote this by writing $Loves(x, y)$.



a. $\exists x \in P.\ Loves(x, x)$

b. $\forall y \in P.\ \exists x \in P.\ Loves(x, y)$

c. $\exists x \in P.\ \forall y \in P.\ Loves(x, y)$

d. $\exists x \in P.\ \forall y \in P.\ (x \neq y \to Loves(x, y))$

e. $\forall x \in P.\ \forall y \in P.\ (x \neq y \to (Loves(x, y) \lor Loves(y, x)))$

f. $\forall x \in P.\ \forall y \in P.\ (x \neq y \to (Loves(x, y) \leftrightarrow \neg Loves(y, x)))$

8. Translating English into First-Order Logic II

Translate each statement into first-order logic, given these predicates: $Dog(x)$ says that x is a dog, $Robot(x)$ says that x is a robot, and $Loves(x, y)$ says that x loves y.

a. Some robot loves exactly one dog. (You can express "there is exactly one thing with a certain property" by saying "there is something with that property, and if something else has that property, then they're the same thing.")

b. There are at least two dogs. (You can talk about multiple objects by nesting quantifiers and, if necessary, checking that the two objects you are looking at are not the same. See the Guide to Logic Translation for more on this.)

9. Negating Statements in First-Order Logic

Negate each of these first-order logic formulas below. The only negations your final formula should have are direct negations of predicates. For example, the negation of the formula $\forall x.\ (P(x) \to \exists y.\ (Q(x) \land R(y)))$ could be found by pushing the negation from the outside inward as follows:

$$\neg(\forall x.\ (P(x) \to \exists y.\ (Q(x) \land R(y))))$$
$$\exists x.\ \neg(P(x) \to \exists y.\ (Q(x) \land R(y)))$$
$$\exists x.\ (P(x) \land \neg(\exists y.\ (Q(x) \land R(y))))$$
$$\exists x.\ (P(x) \land \forall y.\ \neg(Q(x) \land R(y)))$$
$$\exists x.\ (P(x) \land \forall y.\ (Q(x) \to \neg R(y)))$$

Show every step of the process of pushing the negation into the formula.

a. $\exists k.\ \big(Coder(k) \land Athlete(k) \land Painter(k)\big)$

(Hint: Add parentheses to make the inside statement look more like a basic form.)

b. $\forall t.\ \big(Leafy(t) \land Thorny(t) \to Plant(t)\big)$

c. $\exists r.\ \big(Silly(r) \leftrightarrow \neg Serious(r)\big)$

d. $\exists u.\ \Big(Unicorn(u)\Big) \to \exists h.\ \Big(Horse(h) \land Magical(h)\Big)$

e. $\forall x.\ \Big(Person(x) \to \exists y.\ \exists z. \big(CanJuggle(x, y) \land \neg CanJuggle(x, z)\big)\Big)$

f. $\forall p.\ \Bigg(Person(p) \to \Big(\exists q. \big(Person(q) \land TallerThan(p, q)\big)\Big) \lor \Big(\exists q. \big(Person(q) \land TallerThan(q, p)\big)\Big)\Bigg)$

Key tips for negations:

- Go slowly, one step at a time.
- Understand the parentheses in the formula before you start the process.
- If you have a complicated expression, replace it with a symbol and negate it later.