Propositional & First-Order Logic

CS 103ACE Day 3 – 4/12/24

Introduce yourself to someone new and work on #1 together!

Today's learning goals:

- Understand and negate propositional connectives
- Negate more complex expressions
- Understand basic expressions involving first-order quantifiers

Connective	Meaning	Negation(s)
$A \wedge B$	A and B (looks like an A)	
$A \vee B$	A or B (looks like a shrug)	
$\neg A$	Not A	
$A \rightarrow B$	Implication: if A, then B	
$A \leftrightarrow B$	Biconditional: if A then B, and if B then A	
$\top$	True	
$\bot$	False	

Connective	Meaning	Negation(s)
$A \wedge B$	A and B (looks like an A)	$\neg A \vee \neg B$ $A \rightarrow \neg B$
$A \vee B$	A or B (looks like a shrug)	$\neg A \wedge \neg B$
$\neg A$	Not A	A
$A \rightarrow B$	Implication: if A, then B	$A \wedge \neg B$
$A \leftrightarrow B$	Biconditional: if A then B, and if B then A	$A \leftrightarrow \neg B$ $\neg A \leftrightarrow B$
$\top$	True	$\bot$
$\bot$	False	$\top$

Announcements

- Enrollment codes sent out via email! Enroll by 5 pm next Friday
- Office hours schedule for week 3 posted on website
- Looking for a problem set partner? Post on Slack or Ed!

Negating Propositional Formulas

- Go one step at a time
- Order of operations: what is the most important, top-level, or outermost expression in the formula? Negate that one using the table

First-Order Logic I Tips

"All *As* are *Bs*"

$$\forall x.\ (A(x) \rightarrow B(x))$$

"Some *As* are *Bs*"

$$\exists x.\ (A(x) \wedge B(x))$$

- To evaluate "for all x…", try substituting every possible object in for "x"
  - If the inner statement is ever false, the entire statement is false
  - Always true in an empty world
- To evaluate "there exists x…", , try to find just one object that you can substitute in for "x"
  - If the inner statement is ever true, the entire statement is true
  - Always false in an empty world

Post-section recommendations

- Try to memorize the negations for propositional connectives, especially A → B.
- Extra practice problems: everything involving nested quantifiers and logic translations will be covered in today's lecture.
- Start Problem Set 2 over the weekend! Check out office hours on Sunday to get a head start.
- Remember to enroll in ACE!