More First-Order Logic

CS 103ACE Day 4 – 4/15/24

Today's learning goals:

- Understand the 4 first-order logic basic forms
- Translate and interpret nested first-order expressions

$$\forall x.\ (A(x) \rightarrow B(x)) \qquad \exists x.\ (A(x) \wedge B(x))$$

$$\forall x.\ (A(x) \rightarrow \neg B(x)) \qquad \exists x.\ (A(x) \wedge \neg B(x))$$

“All *A*s are *B*s”

$\forall x.\ (A(x) \rightarrow B(x))$

“Some *A*s are *B*s”

$\exists x.\ (A(x) \wedge B(x))$

“No *A*s are *B*s”

$\forall x.\ (A(x) \rightarrow \neg B(x))$

“Some *A*s aren’t *B*s”

$\exists x.\ (A(x) \wedge \neg B(x))$

Problem 7: Interpreting nested statements

Translating:

- Work outside in - one step at a time
- Every time you see a quantifier, replace it:
  - ∃ x. = “there is an x where...”
  - ∀ y. = “for every y...”
- Simplify patterns based on the 4 basic forms

Checking if a statement is true:

- Assign specific examples
  - ∃ => inner statement must be true for <u>some</u> choice
  - ∀ => inner statement must be true for <u>every</u> choice

Problem 8: Translating English to Logic

- Keep asking yourself: Which basic form applies?
  - Can you turn it into a statement about
    everything in some category (universally quantified)
    or some things from the category (existentially quantified)?
- One step at a time
- Give names to any entities mentioned in the statement

Special cases: “exactly one”, “two different”

Problem 9: Negations

Negations are important for indirect proofs involving first-order definitions

- “Pushing the negation inward” demo
- Replace complicated expressions with symbols
- One step at a time
- To check your work: use truth table tool

Our proofwriting toolkit, now with logic

- To show $\forall\ x.\ P(x)$...
  - Have the reader **pick an arbitrary** x.
  - Then, show that P(x) is true.
- To show $\exists\ x.\ Q(x)$...
  - You **give a value** for x.
  - Then, show that Q(x) is true for that value of x.
- To show $A(x) \rightarrow B(x)$...
  - **Assume** that A(x) is true, **Prove** that B(x) is true.
  - Or contrapositive: **Assume** ¬B(x), **Prove** ¬A(x).

Post-section recommendations

- Keep going on Problem Set 2!
- Make sure to look at your Problem Set 1 feedback when it's released
- Recommended extra practice problems for FOL: 4 and 5
- Remember to enroll in ACE!