Functions

CS 103ACE Day 5 – 4/19/24

Agenda:

- Think-pair-share: injections and surjections
- Proofs on function composition
- How to prepare for the midterm

Announcements

- Please ask questions about pset 1 feedback / pset 2!
- Add/drop deadline is tonight! Check your study list!
- Midterm 1 ACE review sessions:
  - Evening of Friday, 4/26
  - Evening of Sunday, 4/28
  - I'm still booking rooms, details will hopefully be finalized in next Monday's section
  - Stanley will also host a Q&A session sometime before the exam (probably Monday 4/29)
- Resources: Academic Coaching, CTL Tutoring

Keep this table open!

	To **_prove_** that this is true…	If you **_assume_** this is true…
$\forall x.\ A$	Have the reader pick an arbitrary x. We then prove A is true for that choice of x.	Initially, **_do nothing_**. Once you find a z through other means, you can state it has property A.
$\exists x.\ A$	Find an x where A is true. Then prove that A is true for that specific choice of x.	Introduce a variable x into your proof that has property A.
$A \rightarrow B$	Assume A is true, then prove B is true.	Initially, **_do nothing_**. Once you know A is true, you can conclude B is also true.
$A \wedge B$	Prove A. Then prove B.	Assume A. Then assume B.
$A \vee B$	Either prove $\neg A \rightarrow B$ or prove $\neg B \rightarrow A$. **_(Why does this work?)_**	Consider two cases. Case 1: A is true. Case 2: B is true.
$A \leftrightarrow B$	Prove $A \rightarrow B$ and $B \rightarrow A$.	Assume $A \rightarrow B$ and $B \rightarrow A$.
$\neg A$	Simplify the negation, then consult this table on the result.	Simplify the negation, then consult this table on the result.

Injectivity

$$\forall a_1 \in A. \forall a_2 \in A. (a_1 \neq a_2 \rightarrow f(a_1) \neq f(a_2))$$

How to prove injectivity 1:

- Pick a_1 and a_2 in A where $a_1 \neq a_2$
- We will show that $f(a_1) \neq f(a_2)$

Read more: Proof Template

Injectivity

$$\forall a_1 \in A.\forall a_2 \in A.(f(a_1) = f(a_2) \rightarrow a_1 = a_2)$$

How to prove injectivity 2:

- Pick a_1 and a_2 in A where $f(a_1) = f(a_2)$
- We will show that $a_1 = a_2$

Read more: Proof Template

Surjectivity

$$\forall b \in B. \exists a \in A. (f(a) = b)$$

How to prove surjectivity:

- Pick b in B
- Show that there exists a in A where f(a) = b
  - To do so, give a value for a
  - Then explain why f(a) = b

Read more: Proof Template

Start thinking about Midterm 1

Midterm 1 is on April 30, in 11 days

More info will be released next week

Covers material from lectures 0-5 and problem sets 1-2 (everything up to and including first-order logic), so **you can start studying now**

Start thinking about Midterm 1

What can you do right now to prepare? **Make a study plan!**

- Plan to study gradually – decide how much time you can spend studying each day
- List all topics from lectures 0-5/psets 1-2, and decide what you are "confident" and "not confident" in
- Spend time learning and practicing your "not confident" topics:
  - do extra ACE problems
  - re-write proofs from lecture or the psets without looking at the solutions
  - come to office hours or ask me questions on Slack

Function Composition
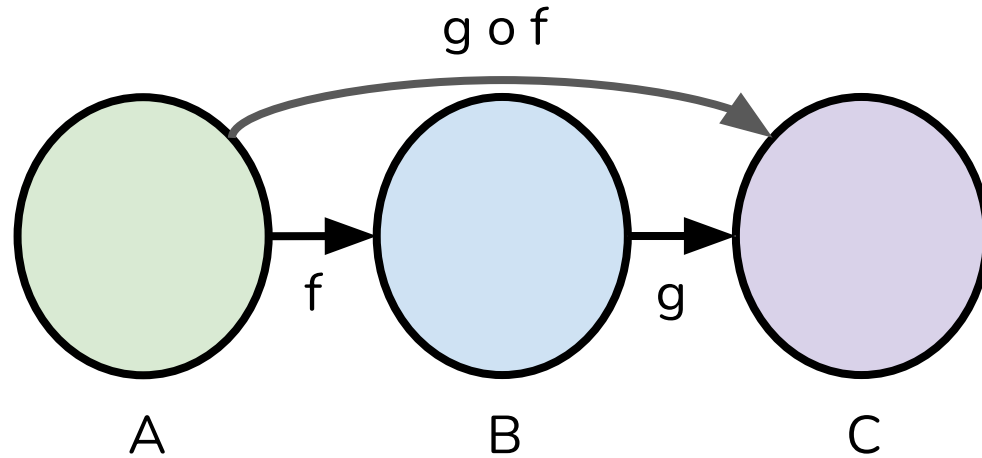
Tip: When substituting into definitions, treat (g o f) as one unit / a function name.
When expanding definitions or applying a function, then you can substitute g(f(x)) for (g o f)(x)

Post-section recommendations

- **Check your study list on Axess before 5pm!**
- Start early on Problem Set 3!
- Make a study plan for Midterm 1!