Set Theory Proofs

CS 103ACE Day 6 – 4/22/24

Agenda:

- Understand the **key strategy for subset proofs**
- Practice using **formal definitions of set operations** to prove things about sets
- (if time) Talk through interpreting function properties

Announcements

- Midterm 1 ACE review sessions:
  - Friday 4/26, 6-8 pm: going over practice problems
  - Sunday 4/28, 6-8 pm: quiet study time + Q&A
  - Location: most likely 320-109
  - Let me know in advance if you want to join on Zoom

	Is defined as...	If you ***assume*** this is true...	To ***prove*** that this is true...
$S \subseteq T$	$\forall x \in S.\ x \in T$	Initially, ***do nothing***. Once you find some $z \in S$, conclude $z \in T$.	Ask the reader to pick an $x \in S$. Then prove $x \in T$
$S = T$	$S \subseteq T \land T \subseteq S$	Assume $S \subseteq T$ and $T \subseteq S$.	Prove $S \subseteq T$. Also prove $T \subseteq S$.
$x \in A \cap B$	$x \in A \land x \in B$	Assume $x \in A$. Then assume $x \in B$.	Prove $x \in A$. Also prove $x \in B$.
$x \in A \cup B$	$x \in A \lor x \in B$	Consider two cases: Case 1: $x \in A$. Case 2: $x \in B$.	Either prove $x \in A$ or prove $x \in B$.
$X \in \wp(A)$	$X \subseteq A$.	Assume $X \subseteq A$.	Prove $X \subseteq A$.
$x \in \{ y \mid P(y) \}$	$P(x)$	Assume $P(x)$.	Prove $P(x)$.

Problem 4 Walkthrough: Power Set Subsets

To prove one set is a subset of another set,

- Have the reader pick an arbitrary element from the **first set**
- Show it is an element in the **second set**

Memorize this! It will be used for almost every subset proof!

Set equality proofs are two subset proofs

This may not be used for subset proofs when we have proven properties relating to sets being subsets of each other. We haven't seen anything like that so far.

Problem 4 Walkthrough: Power Set Subsets

Assume:

Want to show:

$$\wp(A) \cap \wp(B) \subseteq \wp(A \cap B)$$

Problem 4 Walkthrough: Power Set Subsets

Assume:

Want to show:

$$\wp(A) \cap \wp(B) \subseteq \wp(A \cap B)$$

Problem 4 Walkthrough: Power Set Subsets

Assume:

Pick S from $\wp(A) \cap \wp(B)$

Want to show:

S is an element of $\wp(A \cap B)$

Start every subset proof like this!

Problem 4 Walkthrough: Power Set Subsets

Assume:

Pick S from $\wp(A) \cap \wp(B)$

Want to show:

S is an element of $\wp(A \cap B)$

We have **definitions** on each side, so let's expand them.

Problem 4 Walkthrough: Power Set Subsets

Assume:

Pick S from $\wp(A) \cap \wp(B)$

$\rightarrow$ S is an element in $\wp(A)$

$\rightarrow$ S is an element in $\wp(B)$

Want to show:

S is an element of $\wp(A \cap B)$

Problem 4 Walkthrough: Power Set Subsets

Assume:

Pick S from $\wp(A) \cap \wp(B)$

$\rightarrow$ S is an element in $\wp(A)$

$\rightarrow$ S is a subset of A

$\rightarrow$ S is an element in $\wp(B)$

$\rightarrow$ S is a subset of B

Want to show:

S is an element of $\wp(A \cap B)$

Problem 4 Walkthrough: Power Set Subsets

Assume:

Pick S from $\wp(A) \cap \wp(B)$

$\rightarrow$ S is an element in $\wp(A)$

$\rightarrow$ S is a subset of A

$\rightarrow$ S is an element in $\wp(B)$

$\rightarrow$ S is a subset of B

Want to show:

S is an element of $\wp(A \cap B)$

S $\subseteq$ $A \cap B$

It's another subset proof!

Problem 4 Walkthrough: Power Set Subsets

Assume:

Pick S from $\wp(A) \cap \wp(B)$

→ S is an element in $\wp(A)$

→ S is a subset of A

→ S is an element in $\wp(B)$

→ S is a subset of B

Pick x from S

Want to show:

S is an element of $\wp(A \cap B)$

S $\subseteq$ $A \cap B$

x is an element of $A \cap B$

Problem 4 Walkthrough: Power Set Subsets

Assume:

Pick S from $\wp(A) \cap \wp(B)$

$\rightarrow$ S is an element of $\wp(A)$

$\rightarrow$ **S is a subset of A**

$\rightarrow$ S is an element of $\wp(B)$

$\rightarrow$ S is a subset of B

Pick **x from S**

Want to show:

S is an element of $\wp(A \cap B)$

S $\subset$

When we **assume** subset relationships, we get information about **elements of the left-hand set**

x is an element of $A \cap B$

	Is defined as…	If you ***assume*** this is true…	To ***prove*** that this is true…
$S \subseteq T$	$\forall x \in S.\ x \in T$	Initially, ***do nothing***. Once you find some $z \in S$, conclude $z \in T$.	Ask the reader to pick an $x \in S$. Then prove $x \in T$
$S = T$	$S \subseteq T \land T \subseteq S$	Assume $S \subseteq T$ and $T \subseteq S$.	Prove $S \subseteq T$. Also prove $T \subseteq S$.
$x \in A \cap B$	$x \in A \land x \in B$	Assume $x \in A$. Then assume $x \in B$.	Prove $x \in A$. Also prove $x \in B$.
$x \in A \cup B$	$x \in A \lor x \in B$	Consider two cases: Case 1: $x \in A$. Case 2: $x \in B$.	Either prove $x \in A$ or prove $x \in B$.
$X \in \wp(A)$	$X \subseteq A$.	Assume $X \subseteq A$.	Prove $X \subseteq A$.
$x \in \{ y \mid P(y) \}$	$P(x)$	Assume $P(x)$.	Prove $P(x)$.

Post-section recommendations

- Get started on Problem Set 3 if you haven’t already!
- Make some time to study for Midterm 1!
- Take a deep breath, you can do this!!!