1. Reviewing Definitions from Lecture

a. How would you show that a set is not an independent set? (Negate the formal definition.)

b. How would you show that a set is not a vertex cover? (Negate the formal definition.)

c. Explain the Theorem on Friends and Strangers in your own words.

2. Graphs, Sets, and Logic

a. Graphs are defined in terms of sets. Let's explore. Given a graph $G = (V, E)$ and a node $v \in V$, write these sets in set-builder notation:

   (1) "the set containing all edges touching v"

   (2) "the set containing all nodes adjacent to v"

b. Using the predicate $IsPath(a, b, G)$, which says that there is a path between nodes a and b in the graph G, write these statements in first-order logic:

   (1) "G has exactly one connected component"

   (2) "G has more than one connected component"

3. Applying Definitions on Graphs

Prove the following statement:

For any graph $G = (V, E)$: if for any node $v \in V$, the neighborhood set of v is an independent set, then G is triangle-free.

- Note that the following similar statement is untrue: "for any graph $G = (V, E)$, if there is a node $v \in V$ where the neighborhood set of v is an independent set, then G is triangle-free". (One counterexample is the graph in part (b).) What part of the above proof relies on the "for any node $v \in V$" part of the statement?

4. Generating Graphs

For the following problems, draw an example of each type of graph.

a. A graph with one connected component, at least 6 nodes, and an independent set with at least 3 elements. Indicate which nodes of your graph are in your independent set.

b. A graph with two connected components, at least 7 nodes, and a vertex cover with 1 element. Indicate which nodes of your graph are in your vertex cover.

c. A copy of K_4 with edges in two colors, so that there is no monochrome copy of K_3. Recall that K_n is our term for the complete graph with n nodes, and a complete graph is a graph where every pair of nodes is connected by an edge.

d. A graph with at least 5 nodes where the graph contains no cycles, some node has a degree of at least 3, and a different node has a degree of at least 2.