Induction

CS 103ACE Week 6 – 5/3/24

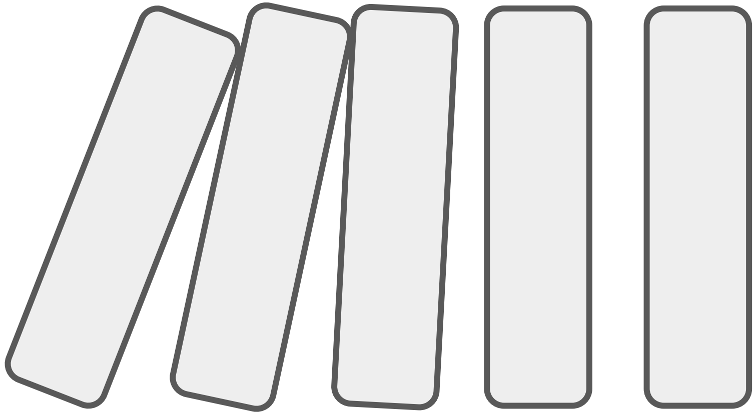
Agenda:

- Induction tips and tricks
- Group proof: recurrence relations
- “Inducting up” and “inducting down”

Announcements

- Things to remember about the midterm
  - Most of your grade (70%+ of it) is still in your hands. There is time to improve!
  - Exams do not reflect whether you belong in CS
  - Exams do not reflect on your worth as a human being
  - I know it’s a tough time of the quarter, you are not alone
- Midterm check-ins next week
  - I will send out a spreadsheet you can use to see your raw score so far
- Next Monday (5/6) during section: small-group feedback
- Clarification on Slack attendance questions

Induction: A real-world comparison



How do we know all the dominoes fall?
1st domino falls
If some domino falls, so does the next

Induction

To prove this statement:

For any natural number n,
$P(n)$ is true

We can use these steps:

1. **Base case:** Show that $P(0)$ is true.
2. **Inductive step:** Show that, for any natural number k,
   if $P(k)$ is true (the **inductive hypothesis**), then $P(k + 1)$ is true

Induction

To prove this statement:

For any natural number n,
$P(n)$ is true

We can use these steps:

1. **Base case:** Show that $P(0)$ is true.
2. **Inductive step:** Pick any natural number k.
   Assume $P(k)$ is true (the **inductive hypothesis**).
   Show that $P(k + 1)$ is true.

Important tips for choosing P(n)

- P(n) should be a predicate: a statement that can be true or false
  - <u>P(n) is not a number</u>
  - P(n) can be an equation itself, but shouldn't be used in equations
- You should be able to plug in a number for n
  - Try crossing out every "n" and replacing it with something like 0 or 103
  - <u>P(n) should not introduce n as a new variable</u>. Treat n like it already has a specific value!

Problem 1. Induction Walkthrough

1. **Restate the theorem with a predicate** P(n).
   a. Often: the exact theorem, crossing out “for all n…”
2. State the **base case** (show P(__) is true) and show it.
3. State the **inductive hypothesis** (pick a k and assume P(k) is true)
4. State the **inductive step goal** (show P(k + __) is true) and show it.
5. **Conclude** that P(n) is true for all natural numbers!

Refreshing on universal vs. existential

	a universal statement “for all unicorns u that are abc, u is xyz.”	**an existential statement** “there is a unicorn u that is abc and xyz.”
assuming		
proving		

Refreshing on universal vs. existential

	a universal statement “for all unicorns u that are abc, u is xyz.”	**an existential statement** “there is a unicorn u that is abc and xyz.”
assuming	Don’t do anything! If you find a unicorn that is abc, you can say that unicorn is xyz.	You can introduce a unicorn into your proof that is both xyz and abc.
proving	Ask the reader to pick a unicorn u that is abc. Show that u is xyz.	You need to come up with a specific value. Then, show that value/object is a unicorn, abc, and xyz.

Building up/down/neither: check the predicate

In induction, the overall theorem is always universal!

"Equation" P(k): something = something

Start with one side of P(k + 1), get to the other

"Build up" P(k): There exists... → P(k + 1): There exists...

Start with the thing we know exists from P(k)

"Build down" P(k): For all... → P(k + 1): For all...

Ask the reader to pick the subject of P(k+1)

Post-section recommendations

- Please let me know how I can support you!
- Keep these important induction strategies in mind as you start Problem Set 5!