Language Concatenation, Kleene Star

CS 103ACE Day 11 – 5/13/24

Agenda:

- Understand the **intuition** and **formal definition** of language concatenation, language powers, and Kleene star
- Strategies for proofs on languages

Announcements

- Week 7 Office Hours and Midterm Review Session times posted on ACE course website

Regular languages & closure properties

Regular languages: languages that we can build a DFA or NFA for

- Why do DFAs and NFAs have equivalent “computing power”?
  - Every DFA is an NFA
  - NFAs can be converted into DFAs via the subset construction (we won’t be talking about this today)
- Regular languages are **closed** under set operations
  - In other words, doing the operation produces another regular language

(What do nonregular languages look like? Stay tuned!)

Language concatenation

- Intuition: $\mathbf{L^n}$ contains all strings made of **n** strings from L **stuck together**
- Formal definition (use this when writing proofs!)

$$L_1 L_2 = \{w \mid \exists x \in L_1.\ \exists y \in L_2.\ w = xy\}$$

$$L^0 = \{\varepsilon\} \qquad L^{n+1} = LL^n$$

Problem 4. The Kleene star

- Operation on a language
- Intuition: **L*** contains all strings made of elements from the original language **L** in any combination/order
- Formal definition (use this when writing proofs!)

$$L_1 L_2 = \{w \mid \exists x \in L_1.\ \exists y \in L_2.\ w = xy\}$$

$$L^0 = \{\varepsilon\} \qquad L^{n+1} = LL^n$$

$$L^* = \{w \mid \exists n \in \mathbb{N}.\ w \in L^n\}$$

Problem 5. Proofs on Languages

Key idea: Languages are just sets, so all our set proofwriting rules still apply to languages!