Nonregular Languages, Myhill-Nerode

CS 103ACE Day 12 – 5/17/24

Learning goals:

- Understand the concepts of distinguishability, distinguishing sets, and nonregular languages
- Be able to apply the Myhill-Nerode Theorem to show a language is not regular

Announcements

- Midterm 2 prep
  - Lectures 6-13 review slides on ACE website
  - Let me know how I can help! I still have Calendly slots open
  - ACE review sessions: 11am to 1pm on 5/18, 7 pm to 9 pm on 5/19, in **Thornton 211** with Zoom option
- Withdrawal / change of grading basis deadline: next Friday 5/24
  - CS 103 grade calculator
  - Please come talk to me if you are worried about your 103 grade
  - "Should I withdraw?"
  - re: ACE attendance, **if you have been in touch with me**, you have no need to worry about the attendance requirement

Nonregular languages

- **Regular languages** can be represented with a DFA, NFA, or regex
- **Nonregular languages** cannot: they need “infinite memory”

Notes on “infinite”:

- All strings have finite length
- All finite languages are regular
- Some infinite languages are regular, and some infinite languages are not regular
  - What’s an example of an infinite regular language?

Distinguishability

Two strings x and y are **distinguishable** relative to a language L when

$$\exists w \in \Sigma^*. \ (xw \in L \leftrightarrow yw \notin L).$$

We write this as $\mathbf{x \not\equiv_L y}$

> Important: we usually use the slash to mean "not", but the slash goes through the ≡ when the strings **are** distinguishable

Intuition: they represent different states within a DFA

What would you have to show for two strings to be distinguishable?

What would it mean for two strings to **not** be distinguishable?

Distinguishing Sets

A set of strings is a **distinguishing set** for a language L when

$$\forall x \in S.\ \forall y \in S.\ (x \neq y \to x \not\equiv_L y)$$

Intuition: all possible pairs of strings from the set are distinguishable.

What would you have to show for a set to be a distinguishing set?

What would it mean for a set to **not** be a distinguishing set?

I have a mystery language L. What's the smallest distinguishing set?

Myhill-Nerode

Today's big takeaway:

To prove a language is not regular, use Myhill-Nerode.

If you can find an infinite distinguishing set for a language, that language is not regular!

Like any existential proof, finding the set can be difficult.

- Tip: try simple sets with a nice structure

Post-section suggestions

- Midterm 2 important concepts to practice:
  - Knowing set theory proofs really well
    - What do you do to show one set is a subset of another?
    - What do you do to show two sets are equal?
  - Interpreting new first-order definitions for functions, graphs, etc. and applying these definitions to problems
    - What do you do to show a universal statement? An existential statement? What about when you assume?
  - Setting up a proof by induction
  - Applying the pigeonhole principle
- Let me know how you're doing!